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ANALYSIS OF THE EFFECT OF THE TEMPERATURE VARIATION ON TRANSISTOR PARAMETERS AND GAIN**

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The variations of the hybrid h-parameters and the gain of the transistor with temperature have been studied by obtaining an analytical expression. The values of the h-parameters are calculated in the entire temperature range 213—473 K with the help of the expression obtained and the calculated values are in good agreement with the experimental data. The gain of the transistor extracted from the obtained values of the parameters is seen to be in excellent agreement with the experimental values.

It is well-known that transistor parameters are dependent on the nature of the material used in the manufacture, and the junction temperature of a transistor in turn affects the properties of the materials. In other words, one can say that the transistor junction temperature is one of the responsible factors for the variation of its parameters. Several workers have tried to control its variation with temperature by using suitable biasing systems. In the present analysis, our aim is to study the variation of the transistor parameters and its gain with temperature, the controlling system not being of interest here. The entire analysis is made with hybrid h-parameters, and a common emitter stage is selected as an example.

Virtually all of the electrical characteristics of the transistor can be ultimately related back to a few fundamental semiconductor parameters, such as minority-carrier concentration, mobility and several universal constants such as electronic charge, Boltzmann's constant, etc. Both the leakage current and the carrier concentration are functions of temperature. With the above background one can conclude that the transistor parameters vary with temperature.

Hybrid h-parameters

In this section, expressions for h_i, h_e, h_f and h_r are derived as functions of temperature with the help of the carrier concentration, mobility and leakage current. The expression for the carrier concentration [1] can be expressed as

$$n_i^2 = pT^3 \exp(-q/T) \tag{1}$$

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where p and q are constants, their values depending on the transistor material (e.g. for Ge, $p = 3.1\ 10^{32}$ and $q = -9101$). By expansion of the exponential term into a power series and neglecting of its lower values terms, Eq. (1) can be approximated as

$$n_i^2 = (p_3T^3 + p_2T^2 + p_1T + p_0) + (p_{-1}T^{-1} + p_{-2}T^{-2} + p_{-3}T^{-3}) \qquad (2)$$

where the p's are constants and can be evaluated from constants p, q and Eq. (1). Through numerical analysis of Eqs (1) and (2), it is found that the contribution of the first bracket towards n_i^2 is much larger than that due to the second bracket. Therefore, for the simplicity of the analysis, the contribution of the second bracket can be ignored. Thus, Eq. (2) reduces to

$$n_i^2 = p_3T^3 + p_2T^2 + p_1T + p_0. \qquad (3)$$

The expression for the leakage current can be expressed as

$$I_{co} = I'_{co} \exp(-r/T - T_0) \qquad (4)$$

where r is a constant and I'_{co} is the leakage current at temperature $T = T_0$. By expansion of the exponential term and neglecting of the lower-value term, Eq. (4) can be approximated as

$$I_{co} = R_3T^3 + R_2T^2 + R_1T + R_0 \qquad (5)$$

where the R's are constants and can be calculated *via* constants related to the leakage current.

The transistor parameter h_{oe} is proportional to conductivity, i.e. $h_{oe}\alpha$ conductivity $(\sigma)\alpha$ mobility $(\mu)x$ carrier concentration. Therefore, $h_{oa}\alpha\mu n_i$. At the same time, we know that [2] $\mu\alpha T^{3/2}$. Thus, from Eq. (3), we have

$$h_{oe}\alpha T^{3/2}(p_3T^3 + p_2T^2 + p_1T + p_0) \qquad (6)$$

With the help of Eq. (6) and experimentally observed curve 3 between $(h_{oe,T}/h_{oe,N})$ and temperature T, one can express the normalized value of h_{oe} as

$$h_{oe,T}/h_{oe,N} = a_0T^3 + b_0T^2 + c_0T + d_0 \qquad (7)$$

where $h_{oe\,T}$ is the value of parameter h_{oe} at a temperature T, $h_{oe,N}$ is the same at normal temperature, i.e. at the reference temperature, and a_0, b_0, c_0 and d_0 are constants.

We know that in a transistor, the collector current, base current and emitter current are all functions of the leakage current. Therefore, *via* Eq. (5) as well as the experimentally observed variation of the normalized value of the transistor parameter h_{ie} with temperature [3], a relation similar to Eq. (7) can also be assigned to h_{ie}:

$$h_{ie,T}/h_{ie,N} = a_iT^3 + b_iT^2 + c_1T + d_i. \qquad (8)$$

Similarly, one can write for h_{fe} and h_{re}

$$h_{fe,T}/h_{fe,N} = a_f T^3 + b_f T^2 + c_f T + d_f . \qquad (9)$$

$$h_{re,T}/h_{re,N} = a_r T^3 + b_r T^2 + c_r T + d_r . \qquad (10)$$

As stated above, the a's, b's, c's and d's are constants and can be calculated with the help of the physical constants related to the transistor under study, but this is very complicated work. To avoid the complications, the constants are calculated from an experimental curve [3]. The four different temperatures $T = 213$ K, 253 K, 373 K and 433 K are taken to find out the constants a's, b's, c's and d's. From the values of these constants, the transistor parameters h_{oe}, h_{ie}, h_{fe} and h_{re} are calculated in the entire temperature range 213–473 K.

Gain of the transistor

The gain G of the transistor is defined as

$$G_T = \frac{h_{fe,T} R_L}{h_T R_L + h_{ie,T}} \qquad (11)$$

where $h_T = h_{ie,T}, h_{oe,T} - h_{fe,T} h_{re,T}$, R_L is the load resistance and suffix T represents the temperature at which the gain of the transistor is to be calculated. It is necessary to state here that the temperature variation of the load resistance has been ignored due to its very small value and also for the simplicity of the calculations.

On substitution of the value of the h-parameters as in Eqs (7–10) into Eq. (11), the expression for the gain G_T reduces to

$$G_T = \frac{a'_f T^3 + b'_f T^2 + c'_f T + d'_f}{A_6 T^6 + A_5 T^5 + A_4 T^4 + A_3 T^3 + A_2 T^2 + A_1 T + A_0 + a'_i T^3 + b'_i T^2 + c'_i T + d'_i} \qquad (12)$$

where

$$\begin{aligned}
a'_f &= h_{fe,N} R_L a_f, \quad b'_f = h_{fe,N} R_L b_f, \quad c'_f = h_{fe,N} R_L c_f, \quad d'_f = h_{fe,N} R_L d_f, \\
A_6 &= A a_i a_0 - B a_r a_f \\
A_5 &= A(a_i b_0 + b_i a_0) - B(a_r b_f + b_r a_f) \\
A_4 &= A(c_i a_0 + b_i b_0 + a_i c_0) - B(c_r a_f + b_r b_f + a_r c_f) \\
A_3 &= A(d_i a_0 + c_i b_0 + b_i c_0 + a_i d_0) - B(d_r a_f + c_r b_f + b_r c_f + a_r d_f) \\
A_2 &= A(d_i b_0 + c_i c_0 + b_i d_0) - B(d_r b_f + c_r c_f + b_r d_f) \\
A_1 &= A(d_i c_0 + c_i d_0) - B(d_r c_f + c_r d_f) \\
A_0 &= A d_i d_0 - B d_f d_r \\
A &= h_{ie,N} h_{oe,N} R_L, \quad B = h_{re,N} h_{fe,N} R_L \\
a'_i &= h_{ie,N} a_i, \; b'_i = h_{ie,N} b_i, \; c'_i = h_{ie,N} c_i \text{ and } d'_i = h_{ie,N} d_i .
\end{aligned}$$

Equation (12) can be further simplified to

$$G_T^{-1} = \frac{B_6T^6 + B_5T^5 + B_4T^4 + B_3T^3 + B_2T^2 B_1T + B_0}{a_f'T^3 + b_f'T^2 + c_f'T + d_f'} \tag{13}$$

where $B_6 = A_6$, $B_5 = A_5$, $B_4 = A_4$, $B_3 = A_3 + a_i'$, $B_2 = A_2 + b_i'$, $B_1 = A_1 + c_i'$ and $B_0 = A_0 + d_i'$.

On the further simplification of Eq. (13), one gets

$$G_T^{-1} = M_3T^3 + M_2T^2 + M_1T + M_0 + \frac{N_2T^2 + N_1T + N_0}{a_f'T^3 + b_f'T^2 + c_f'T + d_f'} \tag{14}$$

where the M's and N's are constants, given by the relations below:

$$a_f'M_3 = B_6,\ a_f'M_2 + b_f'M_3 = B_5,\ a_f'M_1 + b_f'M_2 + c_f'M_3 = B_4,$$
$$a_f'M_0 + b_f'M_1 + c_f'M_2 + d_f'M_3 = B_3,\ N_2 + b_f'M_0 + c_f'M_1 + d_f'M_2 = B_2$$
$$N_1 + c_f'M_0 + d_f'M_1 = B_1,\ N_0 + d_f'M_0 = B_0.$$

Thus, *via* calculation of these constants, the gain of the transistor can be calculated is the entire temperature range 213–473 K.

Results and discussion

The experimental values of the h-parameters at four different temperatures, which have been used to calculate the constants a's, b's, c's and d's, are listed in Table 1 and the obtained value of these constants are listed in Table 2. From these

Table 1

The experimental values of the h-parameters at four different temperatures, which have been used to calculate the constants a's, b's, c's and d's

Temperature, K	$\frac{h_{ie,T}}{h_{ie,T}}$	$\frac{h_{oe,T}}{h_{oe,T}}$	$\frac{h_{re,T}}{h_{re,T}}$	$\frac{h_{fe,T}}{h_{fe,N}}$
213	0.70	0.85	0.65	0.85
253	0.845	0.94	0.845	0.94
373	1.48	1.32	1.58	1.18
433	2.0	1.80	2.40	1.33

The normal (reference) temperature is 20°, i.e. 293 K
$h_{ie,N} = 2.894 \cdot 10^3$ ohm
$h_{oe,N} = 9.599 \cdot 10^{-3}$ ohm
$h_{re,N} = 9.607 \cdot 10^{-4}$
$h_{fe} = 97.09$
$R_L = 5 \cdot 10^3$ ohm (assumed)

Table 2

The values of constants a's, b's, c's and d's obtained with the help of the experimental values of the h-parameters listed in Table 1 and the expressions reported in Eqs (7–10)

Parameters	a	b	c	d
h_{ie}	$3.7238 \cdot 10^{-8}$	$-2.076 \cdot 10^{-5}$	$7.233 \cdot 10^{-3}$	-0.2556
h_{oe}	$9.1515 \cdot 10^{-8}$	$-7.413 \cdot 10^{-5}$	$2.134 \cdot 10^{-2}$	-1.252
h_{re}	$1.550 \cdot 10^{-7}$	$-1.222 \cdot 10^{-4}$	$3.650 \cdot 10^{-2}$	-3.081
h_{fe}	$1.972 \cdot 10^{-8}$	$-1.810 \cdot 10^{-5}$	$7.468 \cdot 10^{-3}$	-0.1102

constants, the normalized values of the h-parameters, i.e. $h_{ie,T}/h_{ie,N}$, $h_{oe,T}/h_{oe,N}$, $h_{re,T}/h_{re,N}$ and $h_{fe\,T}/h_{fe,N}$ have been calculated in the entire temperature range 213–473 K; the results are shown in Figs 1–4. The normal or reference temperature is taken to be 20°, i.e. 293 K. The values of the h-parameters at the reference temperature (293 K) and the used value of the load resistance R_L are also listed in Table 1; *via* calculation of the values of the constants in Eq. (14) related to the gain, the transistor gain G has been calculated in the entire temperature range 213–473 K with the help of Eq. (14), and the value obtained is shown in Fig. 5.

From Figs 1–4 it can be concluded that the expression obtained in Eqs (7–10) gives excellent agreement between the calculated and experimental values of the h-parameters. From these equations, one can also calculate the normalized

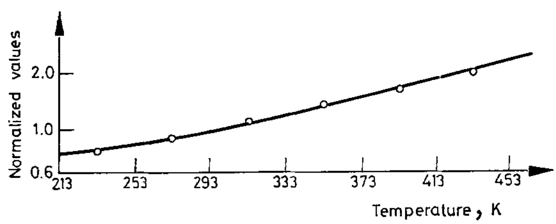

Fig. 1. The normalized value of h_{ie} in the temperature range 213–473 K, obtained *via* Eq. (8). The solid line shows the calculated values and the circles are the experimental points

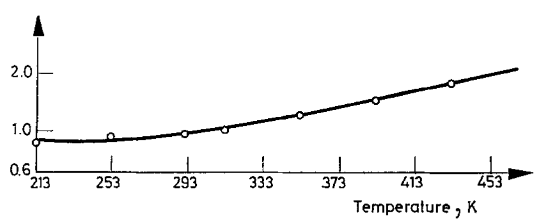

Fig. 2. The normalized value of h_{ce} in the temperature range 213–473 K, obtained *via* Eq. (7). The solid line shows the calculated values and the circles are the experimental points

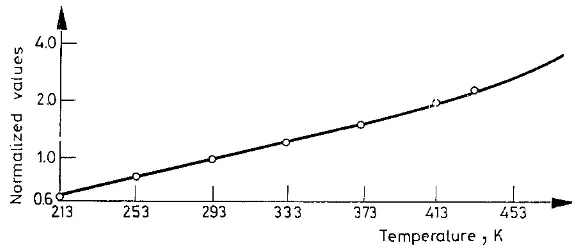

Fig. 3. The normalized value of h_{re} in the temperature range 213—473 K, obtained *via* Eq. (10). The solid line shows the calculated values and the circles are the experimental points

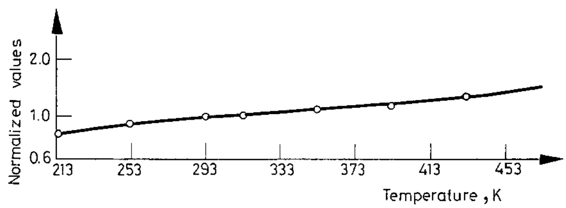

Fig. 4. The normalized value of h_{fe} in the entire temperature range 213—473 K, obtained *via* Eq. (9). The solid line shows the calculated values and the circles are the experimental points

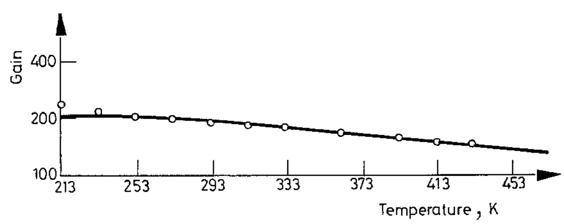

Fig. 5. The gain of the transistor in the entire temperature range 213—473 K. The solid line represents the calculated values and the circles represent the experimental points

values of the *h*-parameters (h_{ie}, h_{fe}, h_{oe} and h_{re}) at any temperature. It can also be concluded that, to control the variation of these parameters with temperature, one has to add a system displaying an opposite nature of variation, i.e. the temperature stability in a transistor can be maintained using a thermistor or diode, which is in agreement with the previous findings [4].

From Fig. 5, it can be concluded that the gain decreases with the increase of temperature, which is an experimental fact. It can also be seen that the agreement between the calculated and experimental values of the gain (the experimental value of gain means that calculated with the help of experimental data) is excellent in the entire temperature range of study. Thus, one can calculate the gain of a transistor at any required temperature by means of Eq. (14). In the present work, the constants *a*, *b*, *c* and *d* are calculated from the experimental data, but these constants can also be calculated with the help of physical constants. *Via* these expressions, one can also study the variation of other parameters.

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RÉSUMÉ — On a étudié, à l'aide d'une expression analytique obtenue pour le but poursuivi, la variation des paramètres *h* hybrides et l'amplification du transistor avec la température. On a calculé, à l'aide de cette expression, les paramètres *h* dans tout l'intervalle des températures comprises entre 213 et 473 K. Les valeurs calculées sont en bon accord avec les données d'expériences. L'amplification du transistor, calculée à partir des valeurs obtenues, est en excellent accord avec les valeurs d'expériences.

ZUSAMMENFASSUNG — Die Änderung des Hybrid-Parameters und die Verstärkung des Transistors mit der Temperatur wurden untersucht, indem ein analytischer Ausdruck hierfür erhalten worden ist. Die Werte der *h*-Parameter wurden im ganzen Temperaturbereich von 213 bis 473 K mit Hilfe des erhaltenen Ausdrucks berechnet und die berechneten Angaben sind in guter Übereinstimmung mit den Versuchsergebnissen. Die aus den erhaltenen Parameterwerten errechnete Transistorverstärkung stimmt mit den Versuchswerten sehr gut überein.

Резюме — Было изучено изменение с температурой смешанных *h*-параметров и усиления транзистора с помощью полученного для них аналитического выражения. Значения *h*-параметров были вычислены во всей температурной области 213—473 K на основе полученного выражения и эти значения хорошо согласуются с экспериментальными данными. Усиление транзистора, выделенное из полученных значений параметров, кажется, должно быть в хорошем согласии с экспериментальными значениями.